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Red sprites: Lightning as a fractal antenna

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Abstract. A new and improved model of red sprites is presented. Emphasis is placed in accounting for the puzzling observation of the spatial structure in the red sprite's optical emissions. The model relies upon a horizontal fractal lightning discharge, which generates the EMPs that excites the optical emissions in the lower ionosphere. It is shown that the fractal model may account for the observed sprite's spatially structured optical pattern, while reducing the typical charge threshold to approximately 100 C.

Introduction

Observations of optical emissions at altitudes between 60 to 90 km associated with thunderstorms have been the focus of many recent ground and aircraft campaigns [*Winckler et al.*, 1993; *Sentman et al.*, 1993]. Among the most puzzling aspects of the observations is the presence of fine structure in the emissions [*Winckler et al.*, 1996]. Vertical striations with horizontal size of 1 km or smaller, often limited by the instrumental resolution, are apparent in the red sprite emissions. Even though the emissions have been the subject of several papers, there is yet no theoretical explanation for the presence of such striated emissions. It is the purpose of this letter to provide the first theoretical model that accounts for the presence of fine structure. It will be shown that the model not only accounts for the structure, but simultaneously reduces the required charge for the lightning discharge driving the red sprites, which have a total optical emission of 100 kR, to levels more consistent with observational requirements.

The model presented here is based on electron energization in the lower ionosphere due to lightning induced transient electromagnetic pulses. However, previous considerations of the lightning induced electric fields were based on horizontal [*Milikh et al.*, 1995] or vertical [*Pasko et al.*, 1995, *Rowland et al.*, 1995] dipole electric models, which produce highly homogeneous and smooth electron heating in the lower ionosphere resulting in the absence of internal structure in the optical emissions. The presented model retains the internal spatio-temporal structure of the lightning discharge.

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The modeling of the internal structure of the discharge is based on the well established observations that the lightning channel follows a tortuous path [*Le Vine and Meneghini*, 1978] and possesses fractal structure down to small scale sizes [*Williams*, 1988]. This is especially true of the spider lightning type discharges which have been suggested as associated with red sprites [*Lyons*, 1996]. It was shown [*Williams*, 1988] that intracloud discharges resemble the well known Lichtenberg patterns observed in dielectric breakdown. These patterns have been recently identified as fractal structures of the Diffusion Limited Aggregate (DLA) type with a fractal dimension $D \approx 1.6$ [*Niemeyer et al.*, 1984]. The lightning discharge will radiate as a fractal antenna and, unlike a dipole type of antenna, will generate a spatially non-uniform radiation pattern with regions of high field intensity and regions of low field intensity, producing the fine structure of red sprites and possibly reducing the required discharging electrical charge by an equivalent gain factor. Furthermore, the fractal lightning fits the qualitative model for the generation of red sprites by *Lyons* [1996], which is based on the fact that horizontal discharges of the order of 100 km have been observed in connection with +CG events. The model starts with the initial spider lightning followed by the positive leader towards the ground, which in turns is followed by the positive return stroke. The later acts as a charge put in the center of a Lichtenberg-like figure, i.e. the lightning discharge propagates along the spider channel. Evidence for these types of models have been obtained from a set of measurements of the properties of the discharges in correlation with the sprites. Red sprites seem to be uniquely correlated with positive cloud-to-ground (+CG) discharges, but only some of the +CG discharges actually generate sprites. Time correlation studies of the time delay between the +CG events and the associated sprite have shown that it can reach more than tens, and sometimes hundreds, of msecs [*Lyons*, 1996], suggestive of the time delay required to develop the horizontal intracloud fractal discharge. As mentioned by *Lyons* [1996] the sprite generating storms seems to have dimensions in excess of 100 km. Such large sizes are also required for the generation of the seed long horizontal discharges. The role played by +CG discharges in the sprite generation can be clarified by The similarity between lightning discharges and dielectric breakdown helps to understand the role played by +CG discharges in the sprite gen-

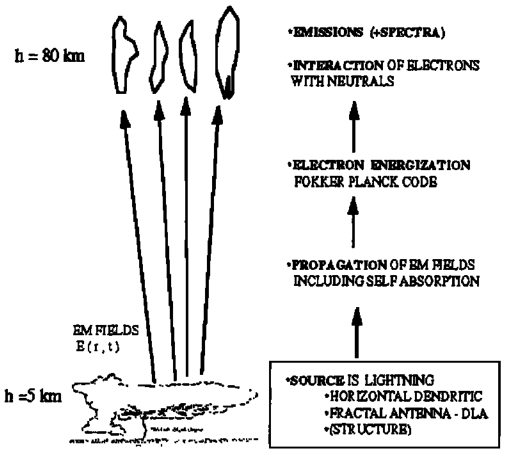

Figure 1. A diagram of the tasks involved in the treatment. From the fractal structure, we compute the fields generated and their interaction with the medium in the lower ionosphere.

eration. Surface dielectric breakdown develops a much intense structure if causes by an immerse positive needle that by a negative one, as revealed by the Lichtenberg figures given by Atten and Saker [1993].

In order to investigate the effect of the fractal nature of the discharges on the ionospheric optical emission pattern, we have constructed a complete model of the generation of sprites as illustrated in Fig. 1. The model starts with the calculation of the transient spatio-temporal electric field pattern $\mathbf{E}(\mathbf{r}, t)$ from a horizontal fractal current structure. As the field propagates into the ionosphere it energize the ambient electrons generating non-Maxwellian electron distribution functions which are calculated with the help of a Fokker-Planck code [*Tsang et al.*, 1991]. The electron distribution function is then used to calculate self-consistently the field self-absorption and the resulting optical emissions. The structuring of the optical emissions is attributed to the highly inhomogeneous lightning induced fields projected in the lower ionosphere, when the discharge internal structure is included in the model.

Modeling lightning as a fractal antenna

A fractal discharge can be modeled as a set of non-uniform distributed current line elements [*Niemeyer et al.*, 1984] $\{\mathbf{x}_n, \mathbf{L}_n, I_n(t)|n = 0, ..., N\}$ with $\mathbf{x}_n$ and $\mathbf{L}_n$ as position and direction of the n^{th} line element respectively. As a current pulse propagates along this horizontal fractal discharge pattern it radiates energy upwards. The fields are computed from the Hertz vector

$$\mathbf{\Pi}(\mathbf{r}, \omega) = \sum_{\{n\}} \widehat{\mathbf{L}}_n \frac{i}{\omega} \int_0^{L_n} I_n(l, \omega) \frac{e^{ik||\mathbf{r}-\mathbf{x}_n - l\widehat{\mathbf{L}}_n||}}{|| \mathbf{r} - \mathbf{x}_n - l\widehat{\mathbf{L}}_n ||} dl. \tag{1}$$

Where $I_n(l, t) = I(t - \frac{(s_n + l)}{v})$ is the current amplitude at the n^{th} line element at time t, s_n is the pathlength to the n^{th} line element, $\mathbf{r}$ is the position at which we measure the fields, ω is the frequency and $k = \frac{\omega}{c}$. Values with the hat^indicate unit vectors.

The intracloud discharge current pulse is taken as $I(t) = I_o(e^{-\alpha t} - e^{-\gamma t}) * (1 + Cos(\eta t))/2$ with $\alpha = 10^3 \, s^{-1}$ and $\gamma = 2 \times 10^5 \, s^{-1}$ assumed from *Uman*, 1987 (Fig 13.6), hence a total discharge of $Q = I_o/\alpha$. The frequency $\eta = 2\pi n_f \, \alpha$ with n_f as the number of oscillations within the current pulse. The spatio-temporal radiation pattern is obtained by inverting the Fourier transformed electric field $\mathbf{E}(\mathbf{r}, \omega) = \nabla \times \nabla \times \mathbf{\Pi}(\mathbf{r}, \omega)$.

We will generate a simple parametrizable model of a two dimensional fractal discharge by combining a set of random walks structured as a developing Cantor set with the following prescription. We take the angular interval $[0, 2\pi]$ of a ring. At the first stage remove the middle third interval $(\frac{1*2\pi}{3}, \frac{2*2\pi}{3})$, leaving the two intervals $[0, \frac{2\pi}{3}]$ and $[\frac{2*2\pi}{3}, 2\pi]$. At the second stage remove the open middle thirds of each of these two intervals leaving four closed intervals of length $2\pi/9$ each, namely the intervals $[0, \frac{1*2\pi}{9}]$, $[\frac{2*2\pi}{9}, \frac{3*2\pi}{9}]$, $[\frac{6*2\pi}{9}, \frac{7*2\pi}{9}]$ and $[\frac{8*2\pi}{9}, 2\pi]$. At every stage repeat the procedure for each of the remaining intervals [*Ott*, 1993]. The very crude fractal electric discharge is created by combining concentric Cantor sets of increasingly finer structure as the radius is increased. The midpoint of the remaining sets, representing a current branch, are connected by a random walk of average step length $< L > \sim 100m$ [*Uman*, 1987]. A fractal structure is shown in Fig 2. The current is assigned at every branching point by satisfying current conservation.

As the current propagates along the dendritic arms of this horizontal fractal, our lightning discharge model radiates upwards, in the z direction, as a fractal antenna. A convenient measured of the amount of energy radiated in a given direction is defined by an array factor $R(\mathbf{r}) = \alpha \int dt E^2(\mathbf{r}, t)$. One method to generate a spatial structure is to require $\eta \Delta r / c > 2\pi$, i.e., $n_f > 50$. Figure 3 shows a cross-section of the spatial structure in

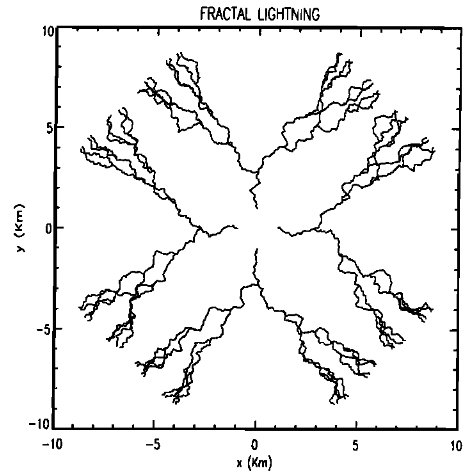

Figure 2. A diagram of the fractal structure generated. A current pulse can propagate along the dendritic arms of this fractal discharge.

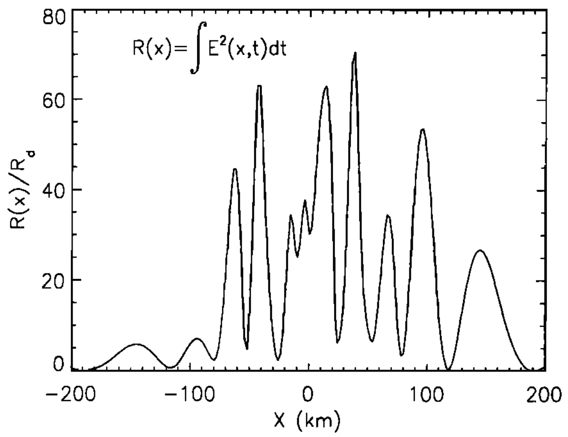

Figure 3. A cross-section along the x direction of the array factor (normalized to the dipole R_d) at $z = 60$ km for the structure of Fig 2.

the array factor along the x direction at $z = 60$ km, normalized by the maximum of the dipole array factor, for the discharge structure shown in Fig. 2 with $n_f = 200$.

EMP absorption and optical emissions

Once we have the fractal discharge structure, we consider the propagation of the lightning induced fields in the lower ionosphere. The fields energize the electrons generating highly non-Maxwellian electron distribution functions which are computed with the help of a Fokker-Planck code that has been developed for the description of ionospheric RF breakdown [*Tsang et al.*, 1991; *Papadopoulos et al.*, 1993; *Milikh et al.*, 1995]. From the electron distribution function we can compute the field absorption and optical emissions produced.

We have assumed that the frequency of the electromagnetic fields satisfy $\omega \ll \omega_B, \nu_e$ and that the magnetic field is perpendicular to the E field ($\theta_o = 90°$). The plasma will reach a steady state in about a collision time, i.e. $t \sim 1/\nu_e$. For the heights of interests, $z < 90$ km, the lowest, hence background, electron-neutral collisional frequency is about a Mhz. Our model and actual measurements [*Uman* 1987] show that the Mhz field amplitudes are well below the 100 kHz field amplitudes. Therefore, we can use the steady state solution from the Fokker-Planck code with the instantaneous electric field $\mathbf{E}^2$ power density. The medium is incorporated in the conductivity $\hat{\sigma}$ and in the dielectric $\hat{\varepsilon}$ tensors [*Gurevich*, 1978], which for the field frequencies and heights of interest, can be considered as independent of time.

Field Self-absorption

As the lightning induced fields propagate in the lower ionosphere, the field changes properties of the medium by heating the electrons while experiencing absorption. The electromagnetic field propagation can be estimated from a nonlinear wave equation [*Gurevich*, 1978; *Taranenko et al.*, 1993; *Inan et al.*, 1996]. In our case, a solution of the applied nonlinear wave equation can be constructed in the ray approximation, given by,

$$E^2(\hat{r}s, t) = \frac{E^2(0, t - \frac{s}{c})}{s^2} e^{-\csc(\chi)\int_0^z \kappa(z,E^2)dz}, \quad (2)$$

where χ is the elevation angle to the point $\mathbf{r} = \hat{r}s$, $\kappa(z, E^2) = \frac{\omega_e^2 \nu_e}{c(\Omega^2+\nu_e^2)}$, $\omega_e^2 = \frac{4\pi n_e e^2}{m}$, and $\Omega = \frac{eB}{mc}$ is the gyrofrequency. The nonlinearity is incorporated through the electron-neutral collision frequency $\nu_e = \nu_e(z, |E|)$, as computed from the electron distribution function [*Tsang et al.*, 1991].

Optical emission

From the spatio-temporal field profile including self-absorption, we can evaluate the optical emission intensity of the red sprites. For simplicity, we consider the excitation of the preferentially red 1st positive ($B^3\Pi_g$) level of molecular nitrogen N_2 which has an excitation energy of 7.35 eV, and a lifetime of 8 μsec. The field and height dependent excitation rate of the 1st positive of N_2 as $\nu_{ex}^{1p} = 4\pi N_{N_2} \int f(\mathrm{v})\mathrm{v}^3 \sigma_{ex}^{1p}(\mathrm{v})d\mathrm{v}$, where σ_{ex}^{1p} is the excitation cross-section. The excitations are then followed by optical emissions where the number of photons emitted per sec per cm^3 is $\nu_{ex}^{1p} n_e$ for an electron density n_e. The intensity of the radiative transition in Rayleighs (column integrated) is given by the integral

$$I = \frac{10^{-6}}{4\pi} \int \nu_{ex}^{1p} n_e dl, \quad (3)$$

carried along the visual path of the detector.

Spatio-temporal emission pattern

The spatio-temporal emission pattern from the fractal discharge structure can now be computed from the field pattern that includes self-absorption. We take a height dependent mid-latitude night time electron den-

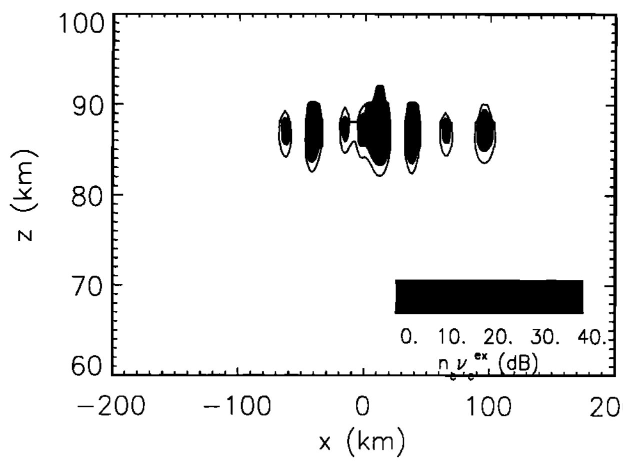

Figure 4. The optical emission pattern for a total discharge of Q=100 C in units of photons per second per cm^3, time averaged over the duration of the discharge, i.e. about a msec.

sity profile and assume a total current of $I_o = 100\,kA$ (total discharge $Q \sim 100$ C). For this case the field is below the ionization threshold thus the electron density is equal to the ambient. In order to compare with observations, we must time average the emission intensity over the time scale of the sprite. The propagation speed during a cloud -to-ground return stroke can reach $\beta = \mathrm{v}/c \sim 0.5$ [Uman, 1987], while the propagation speed of intracloud discharges is at least an order of magnitude lower. The current pulse takes a few millisecond to traverse our fractal, of 10 km in size, with a speed of $\beta = 0.05$. Since the duration of the discharge scales as $t \sim L/\mathrm{v}$, we can keep heating the lower ionosphere at the same rate for a longer time by adjusting the length of the fractal L (a spider discharge can reach 100 km in length [*Lyons*, 1996]).

In our example, the time averaged emission pattern is shown in Fig. 4, for the discharge of $Q = 100$ C, in dBs (base 10 logarithmic scale) with respect to the averaged emission in the computational region. Note how the optical emission pattern shows clear spatial structure. The maximum intensity measured is about 100 kR for an optimal column integration. The optical intensity can be scaled by adjusting I_o and β since $\mathbf{E}^2 \sim I_o^2\,\beta^2$.

Concluding Remarks

A novel model (Fig. 1) of red sprites was presented, that includes that the low altitude lightning has a fractal structure which reflects in the subsequent optical emission pattern. The optical emission pattern depends on the structure of the discharge, but we conjecture that the most relevant parameter in determining the spatial structure of the emissions is the dimension of the self-similar fractal. The other variable that affects the intensity of the fields is the amount of charge discharged.

This model is the first one to account for the fine structure of the sprites. Furthermore, it can generate a sprite with a typical charge value of 100 C, a value significantly lower than required by monopole and dipole models. Assuming the qualitative model of Lyons [1996] we can correlate a charge of $Q \sim 100$ C with the statistics of +CG discharges. Such a discharge seems to occur about 5 % of the time [Uman, 1987] which correlates well with the estimated occurrence rate of sprites [*Sentman*, 1993, *Winckler*, 1993; *Lyons*, 1996].

From the presented model follows that the main body of the sprites is constrained between 80-90 kms in height. The latest observations reveal filaments that can be described as streamers propagating down from the main body of the sprite with a cross-sectional diameter of 100 m or less. Given the nucleated spatial structure in the conductivity produced by the fractal lightning discharge, the streamers would start naturally in the presence of a laminar field. Therefore, a comprehensive model of sprites, that includes the main body produced by the fractal lightning and the subsequent streamer development, has to be developed. Such streamer concept naturally allows for the expansion of the sprite to a wider range in heights. We are currently working on resolving the relevance of this interesting issue.

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